

Computation of LWC and MVD from a Sample Droplet Spectrum

Note 1. This material supplements material found in Chapter I, Sections 1 and 2, of the FAA Aircraft Icing Handbook.

Note 2. All computations and plots presented in this file were generated in the Excel file cldpar.xls.

Assume a sample droplet spectrum from an icing cloud for a given time interval is available in the following form: n sizing bins are defined by the bin boundaries $b_1, b_2, b_3, \dots, b_n, b_{n+1}$, (in microns) so that bin 1 is from b_1 to b_2 , bin 2 is from b_2 to b_3 , ..., bin n is from b_n to b_{n+1} . The bin droplet concentrations for the icing interval have been determined to be c_i droplets per m^3 for $i = 1, \dots, n$.

Let m_i be the midpoint of the i th bin, $i = 1, 2, \dots, n$.

$$m_i = \frac{b_i + b_{i+1}}{2}$$

The liquid water content LWC_i for the droplets in the i th bin can be approximated by using the bin midpoint:

$$LWC_i = c_i \cdot 10^{-12} \cdot \frac{\pi}{6} \cdot m_i^3$$

(Note: It is assumed that water has a density of 1 gram (g) per cubic centimeter (cm^3). 10^{-12} is a conversion factor from cubic microns (μm^3) to cm^3 .)

Then the LWC for the spectrum can be computed by:

$$LWC = \sum_{i=1}^n LWC_i$$

Let pro_i = the proportion of the spectrum LWC that falls in the i th bin.

$$pro_i = \frac{LWC_i}{LWC}$$

Let cum_i = the cumulative proportion of the spectrum LWC that falls in the first i bins.

$$cum_i = pro_1 + \dots + pro_i$$

The median volume diameter (MVD) is defined as the droplet diameter which divides the total water volume in the droplet spectrum such that half the water volume (or liquid water content) is in smaller drops and half is in larger drops. It can be approximated by a linear interpolation with respect to the liquid water content in the $(i+1)$ st bin as follows:

Let i^* = the smallest value of i such that $cum_{i^*} > .5$. Then:

$$MVD = b_{i^*} + \left(\frac{.5 - cum_{i^*-1}}{pro_{i^*}} \right) (b_{i^*+1} - b_{i^*})$$

Table 1 illustrates the computation of the LWC and MVD for a hypothetical instrument.

TABLE 1. COMPUTATION OF LWC AND MVD FOR A HYPOTHETICAL INSTRUMENT

Bin Number	Bin Boundaries	Droplet Conc	Midpoint	LWC _i	pro _i	cum _i
i	b _i (microns)	c _i (per m ³)	m _i (microns)	LWC _i (g/m ³)		
1	10	100000	15	0.00018	0.00058	0.00058
2	20	200000	25	0.00164	0.00538	0.00596
3	30	300000	35	0.00673	0.02214	0.02810
4	40	400000	45	0.01909	0.06274	0.09084
5	50	500000	55	0.04356	0.14320	0.23404
6	60	400000	65	0.05752	0.18909	0.42313
7	70	300000	75	0.06627	0.21786	0.64099
8	80	200000	85	0.06431	0.21143	0.85242
9	90	100000	95	0.04489	0.14758	1.00000
10	100	0	105	0.00000	0.00000	1.00000
	110			LWC = 0.30 g/m³		
					MVD = 73.5 μm	

$$b_1 = 10, b_2 = 20, \dots, b_{11} = 110$$

$$c_1 = 100,000/\text{m}^3, c_2 = 200,000/\text{m}^3, \dots, c_{10} = 0$$

$$m_1 = 15, m_2 = 25, \dots, m_{10} = 95$$

$$\text{LWC}_1 = .00018 \text{ g/m}^3, \text{LWC}_2 = .00164 \text{ g/m}^3, \dots, \text{LWC}_{10} = 0 \text{ g/m}^3$$

$$\text{Pro}_1 = .000581, \text{Pro}_2 = .000538, \dots, \text{Pro}_{10} = 0$$

$$\text{Cum}_1 = .00058, \text{Cum}_2 = .00596, \dots, \text{Cum}_{10} = 1.00000$$

For this example, i* = 7 and

$$MVD = 70 + \left(\frac{.5 - .42313}{.21786} \right) (80 - 70)$$

Figure 1 shows the LWC (on the left vertical axis) and the cumulative proportion of LWC (on the right vertical axis) plotted against the bin right endpoint on the horizontal axis: i.e., LWC₁ and cum₁ vs. d₂, LWC₂ and cum₂ vs. d₃, ..., LWC₁₀ and cum₁₀ vs. d₁₁.

When plotted in this way, the MVD is the point in the diameter on the horizontal axis where the cumulative LWC on the right vertical axis reaches 50 percent of the total.

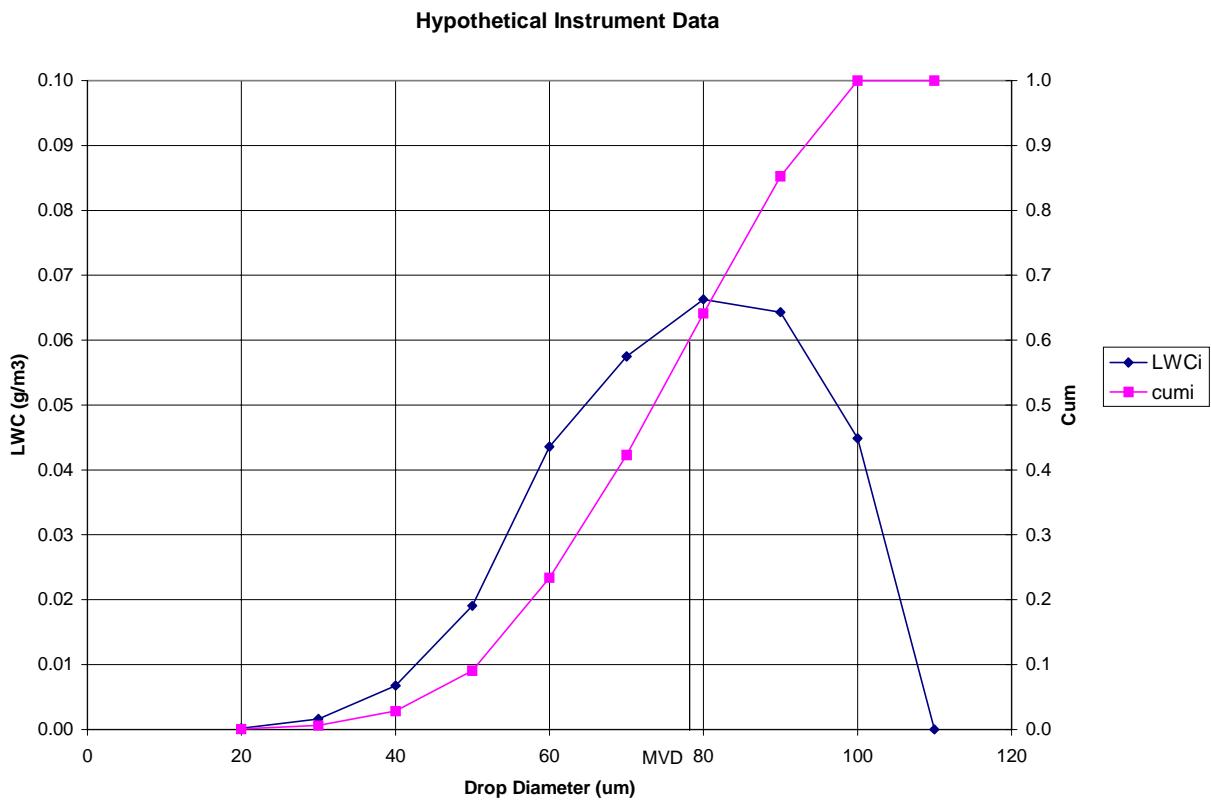


FIGURE 1. HYPOTHETICAL INSTRUMENT DATA

Tables 2, 3, and 4 provide further examples of the computation of LWC and MVD for various instruments or combinations of instruments. The examples are meant to illustrate “real” spectra for instruments in current use. However, the formulae for and calculation of LWC and MVD are exactly the same.

Table 2 illustrates the computation of the LWC and MVD for sample data for a Forward Scattering Spectrometer Probe (FSSP). The spectrum is from an icing wind tunnel. Concentration is typically higher in icing wind tunnels than in flight.

TABLE 2. EXAMPLE OF COMPUTATION OF LWC AND MVD FOR ILLUSTRATIVE DATA FOR AN FSSP.

Bin Number	Bin Boundaries	Droplet					
		b_i (microns)	c_i (per m ³)	m_i (microns)	LWC_i (g/m ³)	pro_i	cum_i
1	2	2.373E+08	3.5	5.327E-03	0.03681	0.03681	
2	5	1.959E+08	6.5	2.817E-02	0.19462	0.23143	
3	8	1.099E+08	9.5	4.934E-02	0.34086	0.57229	
4	11	3.778E+07	12.5	3.864E-02	0.26693	0.83922	
5	14	8.791E+06	15.5	1.714E-02	0.11843	0.95765	
6	17	1.849E+06	18.5	6.130E-03	0.04235	1.00000	

7	20	2.763E-01	21.5	1.438E-09	9.93E-09	1.00000
8	23	1.911E-02	24.5	1.471E-10	1.02E-09	1.00000
9	26	2.184E-03	27.5	2.378E-11	1.64E-10	1.00000
10	29	0.000E+00	30.5	0.000E+00	0	1.00000
11	32	0.000E+00	33.5	0.000E+00	0	1.00000
12	35	0.000E+00	36.5	0.000E+00	0	1.00000
13	38	0.000E+00	39.5	0.000E+00	0	1.00000
14	41	0.000E+00	42.5	0.000E+00	0	1.00000
15	44	0.000E+00	45.5	0.000E+00	0	1.00000
	47			LWC =	0.14 g/m ³	
				MVD =	10.4 μm	

For this example, $i^* = 3$ and

$$MVD = 8 + \left(\frac{.5 - .23143}{.34086} \right) (11 - 8)$$

Table 3 illustrates the computation of the LWC and MVD for sample data for a one dimensional Optical Array Probe (OAP-1D). The spectrum is closely derived from one determined for an icing tanker. Note that in processing the data the OAP bin boundaries have been changed from nominal values to corrected values (14.2 instead of 10.0, 34.0 instead of 30.0, ..., in column B).

TABLE 3. EXAMPLE OF COMPUTATION OF LWC AND MVD FOR ILLUSTRATIVE DATA FOR AN OAP-1D.

Bin Number	Bin Boundaries	Droplet Conc	Midpoint	LWC _i	pro _i	cum _i
i	b _i (microns)	c _i (per m ³)	m _i (microns)	LWC _i (g/m ³)		
1	14.2	3000	24.1	2.199E-05	0.00020	0.00020
2	34.0	11000	43.9	4.873E-04	0.00439	0.00459
3	53.8	22000	63.7	2.977E-03	0.02681	0.03139
4	73.6	16800	83.5	5.121E-03	0.04611	0.07750
5	93.4	15600	103.3	9.004E-03	0.08107	0.15857
6	113.2	12800	123.0	1.247E-02	0.11229	0.27087
7	132.8	10000	142.6	1.518E-02	0.13671	0.40757
8	152.4	7000	162.3	1.567E-02	0.14109	0.54866
9	172.2	4000	182.1	1.265E-02	0.11387	0.66253
10	192.0	3000	201.9	1.293E-02	0.11640	0.77894
11	211.8	2000	221.5	1.138E-02	0.10247	0.88140
12	231.2	800	241.1	5.871E-03	0.05286	0.93426
13	251.0	400	260.8	3.715E-03	0.03345	0.96771
14	270.6	200	280.5	2.311E-03	0.02081	0.98852
15	290.4	90	300.2	1.275E-03	0.01148	1.00000
	310.0			LWC =	0.11 g/m ³	
				MVD =	165.4 μm	

For this example, $i^* = 8$ and

$$MVD = 152.4 + \left(\frac{.5 - .40757}{.14109} \right) (172.2 - 152.4)$$

Table 4 illustrates the computation of the LWC and MVD for sample data for a combined spectrum for an FSSP and an OAP-2D. The spectrum was determined for natural supercooled large droplet conditions in a research flight. Most of the liquid water is in the larger droplets measured. Furthermore, it appears that there are more drops beyond the size range of the OAP-2D, since the LWC is increasing in the last few bins. In processing the data, an interpolation has been performed between the FSSP data and the OAP data starting with the bin from 83 to 98 μm , resulting in the fourteenth bin being 39 μm in size.

TABLE 4. EXAMPLE OF COMPUTATION OF LWC AND MVD FOR ILLUSTRATIVE DATA FOR A COMBINED SPECTRUM FOR AN FSSP AND AN OAP-2D.

Number	Bin i	Bin Boundaries b_i (microns)	Droplet			
			Conc c_i (per m^3)	Midpoint m_i (microns)	LWC _i (g/m^3)	pro _i
1	5	74166.69	6.5	1.066E-05	0.00027	0.00027
2	8	93000.00	9.5	4.175E-05	0.00107	0.00134
3	11	111166.62	12.5	1.137E-04	0.00290	0.00424
4	14	87000.00	15.5	1.696E-04	0.00433	0.00857
5	17	62666.70	18.5	2.078E-04	0.00530	0.01387
6	20	44166.69	21.5	2.298E-04	0.00586	0.01973
7	23	26333.31	24.5	2.028E-04	0.00517	0.02490
8	26	23666.70	27.5	2.577E-04	0.00657	0.03148
9	29	21000.00	30.5	3.120E-04	0.00796	0.03944
10	32	12333.30	33.5	2.428E-04	0.00619	0.04563
11	35	3500.01	36.5	8.911E-05	0.00227	0.04790
12	38	2833.29	39.5	9.143E-05	0.00233	0.05024
13	41	2166.69	42.5	8.709E-05	0.00222	0.05246
14	44	957.88	63.3	1.269E-04	0.00324	0.05569
15	83	36.45	90.0	1.391E-05	0.00035	0.05605
16	98	9.00	105.0	5.455E-06	0.00014	0.05619
17	113	20.55	120.0	1.859E-05	0.00047	0.05666
18	128	10.80	135.0	1.391E-05	0.00035	0.05702
19	143	4.20	157.5	8.592E-06	0.00022	0.05724
20	173	4.05	180.0	1.237E-05	0.00032	0.05755
21	188	12.30	202.5	5.348E-05	0.00136	0.05892
22	218	8.10	225.0	4.831E-05	0.00123	0.06015
23	233	4.05	240.0	2.931E-05	0.00075	0.06090
24	248	8.10	255.0	7.032E-05	0.00179	0.06269
25	263	4.05	270.0	4.174E-05	0.00106	0.06376
26	278	4.05	285.0	4.909E-05	0.00125	0.06501
27	293	16.20	300.0	2.290E-04	0.00584	0.07085
28	308	8.10	315.0	1.326E-04	0.00338	0.07423
29	323	16.20	345.0	3.483E-04	0.00889	0.08312
30	368	8.10	375.0	2.237E-04	0.00571	0.08883

31	383	12.15	390.0	3.774E-04	0.00963	0.09845
32	398	4.05	405.0	1.409E-04	0.00359	0.10205
33	413	16.20	420.0	6.284E-04	0.01603	0.11808
34	428	8.10	435.0	3.491E-04	0.00891	0.12699
35	443	20.25	450.0	9.662E-04	0.02465	0.15163
36	458	8.10	465.0	4.264E-04	0.01088	0.16251
37	473	20.25	480.0	1.173E-03	0.02992	0.19243
38	488	8.10	495.0	5.144E-04	0.01312	0.20555
39	503	4.05	510.0	2.813E-04	0.00718	0.21273
40	518	4.05	525.0	3.069E-04	0.00783	0.22056
41	533	12.15	555.0	1.088E-03	0.02775	0.24830
42	578	0.00	577.5	0.000E+00	0.00000	0.24830
43	578	4.05	585.0	4.245E-04	0.01083	0.25913
44	593	12.15	600.0	1.374E-03	0.03506	0.29419
45	608	4.05	615.0	4.933E-04	0.01258	0.30677
46	623	8.10	630.0	1.060E-03	0.02705	0.33383
47	638	4.05	645.0	5.690E-04	0.01452	0.34835
48	653	8.10	660.0	1.219E-03	0.03111	0.37945
49	668	12.00	697.5	2.132E-03	0.05439	0.43385
50	728	8.10	742.5	1.736E-03	0.04429	0.47814
51	758	8.10	772.5	1.955E-03	0.04988	0.52802
52	788	8.10	832.5	2.447E-03	0.06243	0.59045
53	878	4.20	892.5	1.563E-03	0.03989	0.63033
54	908	7.50	982.5	3.724E-03	0.09502	0.72535
55	1058	4.50	1102.5	3.158E-03	0.08055	0.80590
56	1148	4.20	1177.5	3.590E-03	0.09160	0.89750
57	1208	4.20	1222.5	4.018E-03	0.10250	1.00000
	1238		LWC = 0.04 g/m³		MVD = 770.6 μm	

For this example, $i^* = 51$ and

$$MVD = 758 + \left(\frac{.5 - .47814}{.04988} \right) (788 - 758)$$